ZIMBABWE SCHOOL EXAMINATIONS COUNCIL

General Certificate of Education Advanced Level

CONFIDENTIAL

MARKING SCHEME

NOVEMBER 2019 SESSION

MATHEMATICS

6042/2

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De Moivres – equating imaginary parts M1
– use of identities M1
2 validly obtained A1

1 (a) $Sin3\theta = Sin(\theta + 2\theta)$

$= Sin\theta Cos2\theta + Cos\theta Sin2\theta$ — ~~Use of the identities~~ Correct expression B1

$= Sin\theta(1 - 2Sin^2\theta) + Cos\theta(2Sin\theta Cos\theta)$ — use of at least 1 trig. identities M1

$= Sin\theta - 2Sin^3\theta + 2Sin\theta Cos^2\theta$ ~~Simplifying~~ ~~M1~~

$= Sin\theta - 2Sin^3\theta + 2Sin\theta(1 - Sin^2\theta)$ ~~A1~~

$= 3Sin\theta - 4Sin^3\theta$ Validity obtaining — AG A1
or equivalent method [3]

(b) $\int_0^{\frac{\pi}{3}} Sin^3\theta d\theta = \int_0^{\frac{\pi}{3}} \frac{3Sin\theta - Sin3\theta}{4} d\theta$ — attempt to ~~Sin~~ ~~and Sin3θ~~ M1 ~~B1~~

$= \frac{1}{4}\left[-3Cos\theta + \frac{Cos3\theta}{3}\right]_0^{\frac{\pi}{3}}$ ~~A1 A1~~

$= \frac{1}{4}\left[\left(-\frac{3}{2} - \frac{1}{3}\right) - \left(-3 + \frac{1}{3}\right)\right]$ use of limits M1

$\int Sin\theta Sin^2\theta d\theta$ $= \frac{5}{24}$ A1
[3]

OR equivalent method.

2 $x^2 - 4x + 4 + \frac{1}{16}(k^2 - 6kx + 9x^2) = 25$ Substitution for x or y M1

$16x^2 - 64x + 64 + k^2 - 6kx + 9x^2 = 400$ ~~M1~~

$25x^2 - x(64 + 6k) + k^2 - 336 = 0$ — obtaining quadratic equation A1

$(64 + 6k)^2 - 100(k^2 - 336) = 0$

$4\,096 + 768k + 36k^2 - 100k^2 + 33\,600 = 0$ Use of $b^2 - 4ac$ M1

$-64k^2 + 768k + 37\,696 = 0$

$k^2 - 12k - 589 = 0$ or equiv. A1

$k = \frac{12 \pm \sqrt{144 + 2\,356}}{2}$ Solving for k M1

$= \frac{12 \pm 56}{2}$ ~~Solving~~

$= 31$ or $\frac{-38}{2}$ ~~A1~~

$= 31$ or -19 A1A1
[7]

OR EQUIVALENT METHOD.

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=

3 (a) $g(x) = \frac{2x-1}{x-3}$

$$x-3\overline{)2x-1}\quad 2$$

$$-\frac{(2x-6)}{5}$$

$$\frac{2x-1}{x-3} = 2 + \frac{5}{x-3}$$

use of long division or equivalent method M1

Correct partial fraction A1

[2]

(b) Translation 3 units to the +ve x direction B1

Stretch factor 5 in the y direction B1

Translation 2 units to the +ve y-direction B1

Correct order B1

[4]

(c)

B1 for both asymptotes

B1 for intercepts

B1 for correct shape.

[3]

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4, 5 9.

4 Using similarity

$\frac{1}{r} = \frac{10}{h}$ $h = 10r$

$r = \frac{1}{10}h$ B1

The surface area of water $= \pi r^2$

But $r = \frac{h}{10}$ Expressing A in terms of h

$A = \pi\left(\frac{h}{10}\right)^2$ Expressing A in terms of h. M1

$= \frac{\pi h^2}{100}$

$\frac{dA}{dt} = \frac{dA}{dh} \times \frac{dh}{dt}$

$\frac{dA}{dh} = \frac{2\pi h}{100}$ Chain rule M1

$= \frac{\pi h}{50}$ or $\left(\frac{h}{50}\right)$ A1

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$

$V = \pi\left(\frac{1}{3}\left(\frac{1}{10}h\right)^2 h\right)$

$= \frac{\pi h^3}{300}$

$\frac{dV}{dh} = \frac{3\pi h^2}{300}$ use of Chain rule for $\frac{dV}{dh}$ B1

$h=5$ $= \frac{\pi h^2}{100}$ or $\pi/4$. for correct use of chain rule M1

$\frac{dh}{dt} = 0.1 \times \frac{100}{\pi h^2}$

$= \frac{10}{\pi h^2}$ $\frac{2}{5\pi}$ A1

$\frac{dA}{dt} = \frac{\pi h}{50} \times \frac{10}{\pi h^2}$ use of chain rule

$= \frac{1}{25}$ Correct $\frac{dA}{dt}$ M1A1

$\frac{1}{4} + \frac{1}{6} = \frac{1}{2}$ $\frac{3+2}{12} = \frac{5}{12}$

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$= \frac{1}{5A}$

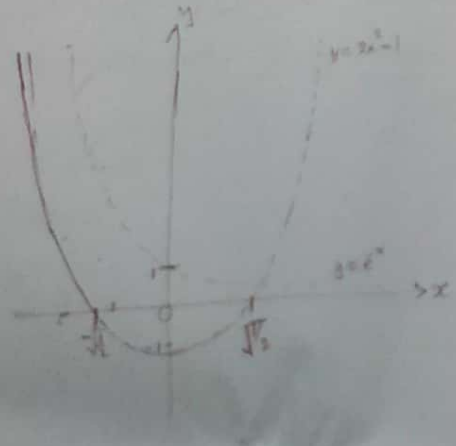
When it is halfway $h = 5$ For substituting $h = 5$

$\frac{dA}{dt} = \frac{1}{5\times5}$

$= \frac{1}{25}$

[9]

5 (a)



B1 one correct graph
B1 for another correct graph } Correct shape
B1 for conclusion one - real root.

$f(x) = e^{-x} - 2x^2 + 1$
$f(x) = 2x^2 - 1 - e^{-x}$

wrong answer, Correct substitution M1 to A0

(b) $f(1) = \cancel{1.10}$ $-0,632120559$ Substitution into the function $f(x)$ M1

$f(0.5) = \cancel{-0.632}$ $1,10653066$ for both correct A1

$f(0.5) \times f(1) < 0$ or equivalent Conclusion A1
[3]

(c) $x_2 = \sqrt{\frac{e^{-1}+1}{2}}$ Correct use of formula M1

$= \cancel{0.6839397}$ $0,827006481$ A1

$x_3 = 0.82700$

$x_4 = 0.84774$

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$x_5 = 0.84509$

$x_6 = 0.84539$

$\therefore$ the root = 0.845 — Correct answer to 3 d.p — A1 [3]

6 (a) $\int_{-2}^{0} \frac{1+2x}{1-2x} dx = \int_{-2}^{0} \left(-1 + \frac{2}{1-2x}\right) dx$ — Use of partial fractions — B1

$= [-x - 1n(1-2x)]_{-2}^{0}$ — Attempt to integrate and correct integral — M1A1

$= 0 - (2 - ln5)$

$= ln5 - 2$ — A1 [4]

(b) (i) $-\frac{d\theta}{dt} \propto (\theta - 20)$ — Using proportionality — M1

$-\frac{d\theta}{dt} = k(\theta - 20)$

$\frac{d\theta}{dt} = -k(\theta - 20)$ — A1 [2]

(ii) $\int \frac{d\theta}{\theta - 20} = \int -k dt$

$ln(\theta - 20) = -kt + c$ — Attempting to integrate and correct integral — M1A1

~~$1n(\theta - 20) = -kt + c$~~ — ~~Attempting to integrate and correct integral~~ — ~~M1~~

when t = 0 $\theta = 60°$

$ln(60 - 20) = k(0) + c$ — Use of initial conditions — M1

$c = ln40$ — A1

$ln(40 - 20) = -5k + ln40$ — Substituting $t=5$ $\theta=40$ — M1

$5k = ln40 - ln20$

$k = 5ln\left(\frac{40}{50}\right)$ — Correct value of k. — A1

$k = \frac{1}{5} ln 2$ or equiv.

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$= \frac{1}{5}\ln 2$ — Correct value of k — B1

$\ln(\theta - 20) = -kt + c$ — ~~Exponentiating~~ — ~~M1~~

Exponentiating — M1

$\theta - 20 = e^{-kt+c}$

$\theta - 20 = e^{c} \cdot e^{-kt}$

$\theta - 20 = e^{\ln 40} \cdot e^{\frac{1}{5}\ln 2}$

$\theta = 40e^{-\frac{t}{5}\ln 2} + 20$ — A1 [8]

OR EQUIV.

(iii) After 10 minutes

$\theta = 40e^{-\frac{10}{5}\ln 2} + 20$ — Sub. $t = 10$ — M1

$= 40e^{-2\ln 2} + 20$

$= 40 \cdot \frac{1}{4} + 20$

$= 30°C$ — A1 [2]

7 (a) For closure $\begin{pmatrix} a & b \\ 2b & a \end{pmatrix} + \begin{pmatrix} c & d \\ 2d & c \end{pmatrix}$ — Addition of two matrix — M1

use of real number wrong method, ~~wrong addition~~ wrong method.

$= \begin{pmatrix} a+c & b+d \\ 2(b+d) & (a+c) \end{pmatrix} \in S$ — A1

Association

$\begin{pmatrix} a & b \\ 2b & a \end{pmatrix} + \left[\begin{pmatrix} c+e & d+f \\ 2d+2f & c+e \end{pmatrix}\right]$ — for showing 3 matrices and proving equality — M1

$= \begin{pmatrix} a+ & (c+e) & b+(d+f) \\ 2b+(2d+2f) & & a+(c+e) \end{pmatrix}$

$= \begin{bmatrix} (a+c)+e & (b+d)+f \\ (2b+2d)+2f & (a+c)+e \end{bmatrix}$

~~Real numbers association~~

$\begin{bmatrix} a+c & b+d \\ 2b+2d & a+c \end{bmatrix} + \begin{pmatrix} e & f \\ 2f & e \end{bmatrix}$

$\left[\begin{pmatrix} a & b \\ 2b & a \end{pmatrix} + \begin{pmatrix} c & d \\ 2d & c \end{pmatrix}\right] + \begin{pmatrix} e & f \\ 2f & e \end{pmatrix}$ — A1

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$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is the identity element — B1

$\begin{pmatrix} -a & -b \\ -2b & -a \end{pmatrix}$ is the inverse — B1

hence it is a group — for conclusion B1 [7]

(b)

	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

B3 - 1 for each wrong entry

for closure B1.

The identity element is 0 — B1

Every element has a corresponding inverse x^*

at least one inverse correct → M1A1 all correct

x	0	1	2	3	4
x^*	0	4	3	2	1

It is associative

provided reasonable attempt at least one working done. M1A1 [9]

8 (a) $\pi_1 \quad r \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = 2$

$r = \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 2\lambda + 2 \\ \lambda + 1 \\ 2\lambda + 1 \end{pmatrix}$

General point of line in parameter form — B1

$\begin{pmatrix} 2\lambda + 2 \\ \lambda + 1 \\ 2\lambda + 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

$= 2\lambda + 2 + 2\lambda + 2 - 4\lambda - 2$ — Use of dot product M1

$= 4\lambda - 4\lambda + 4 - 2$

$= 2$ A1 [3]

OR equiv. method

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(b) (i) $[x + 2y - 2z = 2] \times 2$

$2x - 3y + 6z = 3$

$2x + 4y - 4z = 4$

$2x - 3y + 6z = 3$

$7y - 10z = 1$

$7y = 10z + 1$ — — — Attempting to solve simultaneously for any one of the variables M1

$y = \frac{10z}{7} + \frac{1}{7}$ — — — any 1 correct equation A1

$2x = 3y - 6z + 3$ ~~Any 1 correct~~ ~~A1~~

$x = \frac{3}{2}\left(\frac{10z}{7} + \frac{1}{7}\right) - 3z + \frac{3}{2}$ Solving for a second variable M1

$= \frac{15z}{7} + \frac{3}{14} - 3z + \frac{3}{2}$ ~~Both correct~~ ~~A1~~

$= \frac{-6z}{7} + \frac{12}{7}$ — — — — — — — — — A1

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{-6z}{7} + \frac{12}{7} \\ \frac{10z}{7} + \frac{1}{7} \\ z \end{pmatrix}$ ~~Subt~~ M1

$= \begin{pmatrix} \frac{12}{7} \\ \frac{1}{7} \\ 0 \end{pmatrix} + z\begin{pmatrix} \frac{-6}{7} \\ \frac{10}{7} \\ 1 \end{pmatrix}$ for correct direction vector. A1

~~taking z = 7~~

$r = \begin{pmatrix} \frac{12}{7} \\ \frac{1}{7} \\ 0 \end{pmatrix} + t\begin{pmatrix} -6 \\ 10 \\ 7 \end{pmatrix}$ [6]

(ii) $r = \begin{pmatrix} 2 + 2\lambda \\ 1 + \lambda \\ 1 + 2\lambda \end{pmatrix}$

$\begin{pmatrix} 2 + 2\lambda \\ 1 + \lambda \\ 1 + 2\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} = 3$

$4 + 4\lambda - 3 - 3\lambda + 6 + 12\lambda = 3$ — — — Use of dot product M1

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$\lambda = \frac{-4}{13}$

Correct value of λ A1

$r = \begin{pmatrix} 2 - 2\left(\frac{-4}{13}\right) \\ 1 - \frac{4}{13} \\ 1 + 2\left(\frac{-4}{13}\right) \end{pmatrix}$

~~Correct value of r~~ ~~A1~~

$\begin{pmatrix} \frac{18}{13} \\ \frac{9}{13} \\ \frac{5}{13} \end{pmatrix}$

* for column / or row vector. A1

Point of intersection?

$\frac{1}{13}\begin{pmatrix} 18 \\ 9 \\ 5 \end{pmatrix}$ [3]

(iii) $r = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

$\pi_2 \Rightarrow r \cdot \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix} = 3$

direction vector of L = $\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$

Normal vector to plane $\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$

$= \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$

$= 4 - 3 + 12$

$= 13$

$\sqrt{4+1+4}$

$= \sqrt{9}$

$= 3$

$\sin\theta = \frac{\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}}{\sqrt{9} \cdot \sqrt{49}}$ M1

$= \frac{13}{21}$ A1

$\theta = \sin^{-1}\left(\frac{13}{21}\right)$ M1

$= 38{,}2^\circ$ A1

Use of dot product M1

Alternative method

$\cos\alpha$

$\alpha = 51{,}8^\circ$ M1A1

$\theta = 90 - 51{,}8^\circ$ M1A1

$= 38{,}2^\circ$

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$\sqrt{4 + 9 + 36}$ Correct modulus A1

$= \sqrt{49}$

$= 7$

$Sin\theta = \frac{13}{7\times3}$

$= \frac{13}{21}$ Simplifying M1

$\theta = Sin^{-1}\left(\frac{13}{21}\right)$

$= 38.2°$ to 1 d.p A1
[4]

Accept use of $n = 2$

9 (a) Let n be true for $n = 1$

LHS $ar^{n-1} = a\left(\frac{1-r^n}{1-r}\right)$ RHS — — — for substitution M1

$ar^{1-1} = a\left(\frac{1-r^1}{1-r}\right)$ A1

$a = a\frac{1-r}{1-r}$ A1

$a = a$

Hence true for $n = 1$ — — — — — — for conclusion A1

Let it be true for $n = k$

$\sum_{n=1}^{k} ar^{n-1} = a\left(\frac{1-r^k}{1-r}\right)$ — — — — B1

Let it be true for $n = k + 1$

LHS RHS

Condone use r

$\sum_{n=1}^{k+1} ar^{n-1} = \sum_{n=1}^{k} ar^{n-1} + (k+1)^{th}$ term

$= \sum_{n=1}^{k} ar^{n-1} + ar^k$

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$= a\left(\frac{1-r^k}{1-r}\right) + ar^k$ Use of hypothesis ---M1

$= \frac{a - ar^k + ar^k - ar^k \times r}{1-r}$ Expansion and use of LCM. M1

$\frac{a-ar^k \times r}{1-r}$ ~~Use of LCM~~

$a\left(\frac{1-r^{k+1}}{1-r}\right)$ ---- Validity obtaining $k+1$ ---A1

~~Hence true for $n = k+1$~~

By induction it is true for $n = 1$, $n = k$ and $n = k+1$

Hence true for all values of n Correct concluding statement ---A1

[7]

(b) (i) $\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$

$\frac{r+2-r}{r(r+1)(r+2)}$ Use of LCM ---M1

$= \frac{2}{r(r+1)(r+2)}$ ---A1

[2]

(ii) $\sum \frac{1}{r(r+1)(r+2)} = \frac{1}{2}\sum \frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$ Factoring out $\frac{1}{2}$ seen or implied B1

of partial fractions Mark. word mark.

$(r=1) = \frac{1}{2}\left[\frac{1}{1\times 2} - \frac{1}{2\times 3}\right]$

$+\frac{1}{2}\left[\frac{1}{2\times 3} - \frac{1}{3\times 4}\right]$ Method of differences ----M1

$+\frac{1}{2}\left[\frac{1}{3\times 4} - \frac{1}{4\times 5}\right]$

1 —— 1
1 —— 1
1 —— 1
1 —— 1

Cancelation seen or implied ---M1

A=1 B=-2 B1 C=1

$r = n-1$ $+\frac{1}{2}\left[\frac{1}{(n-1)n} - \frac{1}{n(n+1)}\right]$

$r = n$ $+\frac{1}{2}\left[\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right]$ ~~Grouping~~

A=1/2 B=-1 C=1/2

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$= \frac{1}{2}\left(\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right)$ grouping M1

$= \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$ Validly obtaining answer given A1

[5]

(iii)

$\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)}$

$= \frac{1}{4} - 0$ M1

$= \frac{1}{4}$ A1

for Gabt working $\frac{1}{4}$ (B2) [2]

10 (a) $1 - i$ is a root ⇒ its conjugate is also a root $1 + i$ ——— B1

$(x - 1 + i)$ and $(x - 1 - i)$ are factors $x^2 - 2x + 2$ —— Quadratic factor B1

$$x^2 - 2x + 2 \overline{)x^4 - 4x^3 + 2x - 6}\ \big)\ x^2 - 2x - 3$$

$-[x^4 - 2x^3 + 2x^2]$

$-2x^3 + x^2 + 2x$ Long division ——— M1

$-[-2x^3 + 4x^2 - 4x]$

$-3x^2 + 6x - 6$

$-[-3x^2 + 6x - 6]$

0 0 0 Quotient

$x^2 - 2x - 3$ is another quadratic factor Obtaining the quadratic factor ——A1

$(x - 3)(x + 1) = 0$ Factorisation ——— M1

$x = -1$ or 3

$x = 1 - i, 1 + i, -1$ and 3 ——— for -1 and 3 ——— A1

Complete set. [6]

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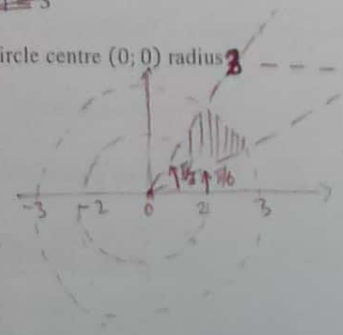
(b) ~~$|z| = 3$~~

Circle centre (0; 0) radius 3 — condone broken lines — B1

~~Correct circles~~



Correct circle, centre (0,0), radius 3 B1

Line segments and arguments B1

~~Shaded region~~ B1

Correct region B1

[4]

(c) Let $z = r(Cos\alpha + iSin\alpha)$ be the root

then $z = 8\sqrt{3} - 8i$ B1

$|z| = 16$ B1

$\theta = -30°$ seen or implied.

~~finding argument and modulus~~

$Z^4 = 16[Cos30° - iSin30°]$

$r^4[Cos4\alpha + iSin4\alpha] = 16[Cos30° - iSin30°]$

$r^4 = 16$ equating / comparison for both M1

$r = 2$

$4\alpha = -30° + 360k$

$\alpha = -7.5° + 90k$ $k = 0; 1; 2; 3$ general argument and $|r|$ A1

rads ~~radians~~

$z_1 = 2[Cos7.5 - iSin7.5°] = 2(0.9914 - 0.1305i)$ for substitution M1

$1.98 - 0.26i$

$Z_2 = 2[Cos82.5° + iSin82.5°] = 0.26 + 1.98i$ ~~any two correct~~

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$Z_3 = 2[Cos172.5° + iSin172.5°] = -1.98 + 0.26i$

$Z_4 = 2[Cos262.5° + iSin262.5°] = -0.26 - 1.98i$ All correct A1 [6]

11 (a) Matrix of operation = HM

Reflection of the x - axis $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ Seen or implied – Reflection matrix B1

$HM = \begin{pmatrix}1 & 2\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$ – Combining matrices M1

Multiplication starts with reflection

$\begin{pmatrix}1 & -2\\0 & -1\end{pmatrix}$

$\begin{pmatrix}1 & -2\\0 & -1\end{pmatrix}\begin{pmatrix}0 & 1 & 3 & 2\\0 & 2 & 3 & 1\end{pmatrix}$ – Method of finding coordinates M1

$\begin{pmatrix}0 & -3 & -3 & 0\\0 & -2 & -3 & -1\end{pmatrix}$

$(0;0)$; $(-3;-2)$; $(-3;-3)$ and $(0;-1)$ Correct coordinates A1 [4]

(b) (i) AB

$\begin{pmatrix}1 & 1 & 0\\1 & 0 & -1\\1 & 1 & 2\end{pmatrix}\begin{pmatrix}1 & -2 & -1\\-3 & 2 & 1\\1 & 0 & -1\end{pmatrix}$

$\begin{pmatrix}-2 & 0 & 0\\0 & -2 & 0\\0 & 0 & -2\end{pmatrix}$

B2 – 1 each wrong entry

(ii) $A^{-1} = -\frac{1}{2}\begin{pmatrix}1 & -2 & -1\\-3 & 2 & 1\\1 & 0 & -1\end{pmatrix}$ for determinant $(-\frac{1}{2})$ B1; for correct entries B1 [4]

(iii) $\begin{pmatrix}1 & 1 & 0\\1 & 0 & -1\\1 & 1 & 2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}3\\5\\7\end{pmatrix}$

~~Premultiplying by the inverse~~

$\begin{pmatrix}x\\y\\z\end{pmatrix} = \frac{-1}{2}\begin{pmatrix}1 & -2 & -1\\-3 & 2 & 1\\1 & 0 & -1\end{pmatrix}\begin{pmatrix}3\\5\\7\end{pmatrix}$

~~Use of inverse~~ premultiplying by the inverse M1

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$= \frac{-1}{2}\begin{pmatrix} 3-10-7 \\ -9+17+5 \\ 3-7 \end{pmatrix}$ Simplification / Multiplying

$= \frac{-1}{2}\begin{pmatrix} -14 \\ 8 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} 7 \\ -4 \\ 2 \end{pmatrix}$ $x = 7$ A1 $y = -4$ A1 $z = 2$ A1 [4]

(c) $x_{n+1} = x_n - \frac{f(x_n)}{f^1(x_n)}$

$\frac{dy}{dx} = 3x^2 - 5$ — derivative B1

$x_1 = 4 - \frac{(4)^3 - 5(4) - 40}{3(4)^2 - 5}$ — Correct structure M1

$= 3.9069767442$ A1

$x_2 = 3.90445096923$

$\therefore x = 3.90$ to 3. S f — c.a.o A1 [4]

12 (a) (i) $\frac{dy}{dx} = y^3 + x^8$

Correct derivative For differentiation B1

$\frac{d^2y}{dx^2} = 3y^2\frac{dy}{dx} + 8x^7$

$\frac{d^3y}{dx^3} = 6y\left(\frac{dy}{dx}\right)^2 + 3y^2\frac{d^2y}{dx^2} + 56x^6$

Any 1 correct term M1 for any 2 correct terms A1

Substitute for $x=0, y=1$ M1

$\frac{dy}{dx} = 1$

$\frac{d^2y}{dx^2} = 3$

for any one correct derivative A1

for all correct derivatives A1

$\frac{d^3y}{dx^3} = 6 + 3(3) = 15$

$f(x) = 1 + x(1) + \frac{x^2}{2}(3) + \frac{x^3}{6}(15)$

for substitution M1

$f(x) = 1 + x + \frac{3x^2}{2} + \frac{5}{2}x^3$ A1 [8]

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(ii) $f(1.2) = 1 + 1.2 + \frac{3}{2}(1.2)^2 + \frac{5}{2}(1.2)^3$ for substitution into his M1

$= 2.2 + 2.16 + 2.88$

$= \cancel{7.24}$ 8.08 A1 [2]

8 17/25

(b) $f(x) = e^{\sqrt{x}}$

$f(4) = e^2$ B1

$f^1(x) = \frac{1}{2}x^{\frac{-1}{2}}e^{\sqrt{x}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$ for attempting Differentiation M1

$f^{11}(x) = \frac{e^{\sqrt{x}}}{2\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} - \frac{1}{4}x^{\frac{-3}{2}}e^{\sqrt{x}}$

$f^1(4) = \frac{e^2}{4}$

$f^{11}(4) = \frac{e^2}{4}\left(\frac{1}{4}\right) - \frac{e^2}{4(8)}$ for Substitution M1

$= \frac{e^2}{32}$ at least one of $f^1(4)$ or $f^{11}(4)$ A1

$= f(x) = e^2 + x\left(\frac{e^2}{4}\right) + \frac{x^2}{2}\left(\frac{e^2}{32}\right)$

$= e^2 + \frac{e^2}{4}(x-4) + \frac{e^2}{64}(x-4)^2 + \cdots$ Substitution into Taylor's series M1 A1 [6]

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